

# Analyzing Chiral Symmetry Breaking in Supersymmetric Gauge Theories

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## Abstract

We compare gap equation predictions for the spontaneous breaking of global symmetries in supersymmetric Yang-Mills theory to nonperturbative results from holomorphic effective action techniques. In the theory without matter fields, both approaches describe the formation of a gluino condensate. With  $N_f$  flavors of quark and squark fields, and with  $N_f$  below a certain critical value, the coupled gap equations have a solution for quark and gluino condensate formation, corresponding to breaking of global symmetries and of supersymmetry. This appears to disagree with the newer nonperturbative techniques, but the reliability of gap equations in this context and whether the solution represents the ground state remain unclear.

## 1 Introduction

Spontaneous breaking of global symmetries in a gauge field theory sets in only when the gauge forces become strong. Accordingly, nonperturbative methods must be brought to bear, and the problem remains only partially understood. Approaches include lattice methods, semiclassical methods, and  $1/N$  expansions. In addition, a simple approach based in Feynman graphs has been used with apparent success for many years. It comes in several closely related forms, including Schwinger–Dyson or gap equations, and “most attractive channel” analyses. These share the common idea that the forces responsible for symmetry breaking can be computed in a loop expansion. The lowest order contribution is typically one-particle exchange, with higher order contributions given by two

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particle irreducible graphs [1, 2]. This can be applied to a most attractive channel force analysis or equivalently the kernel of a gap equation, or the CJT effective potential [1] for composite operators.

Because the coupling must be relatively strong, there is no obvious small parameter in the expansion and this approach has been met with justifiable skepticism. Some evidence in favor of its utility, however, is provided by estimates of the second order contribution to the kernel [3, 4]. When the coupling is just strong enough to trigger the breaking, second order corrections are relatively small (less than 20%). With this check in mind, and in the absence of clearly superior alternatives, gap equations have been widely used in recent years in the study of dynamical electroweak symmetry breaking [5]. Still, their validity remains open to question.

Recent advances [6] in nonperturbative methods for supersymmetric gauge theories provide an independent framework in which to test the validity of the gap equation approach. The new methods lead to conclusions about spontaneous breaking of global symmetries in supersymmetric theories, which can be compared to gap equation results. This letter reports the results of such a comparison, first for the case of  $\mathcal{N} = 1$  supersymmetric Yang-Mills  $SU(N_c)$  theory with no matter fields (pure SQCD), and then for  $\mathcal{N} = 1$  SQCD including matter fields.

## 2 Pure Supersymmetric QCD

The Lagrangian for pure SQCD contains only gluon and gluino degrees of freedom (in Wess-Zumino gauge after eliminating auxiliary fields):

$$\mathcal{L}_{SYM} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} + \frac{\theta g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{\mu\nu a} + \frac{1}{2}\bar{\lambda}^a i\not{D}\lambda^a + \text{gauge fixing} + \text{ghosts} , \quad (1)$$

where the canonically normalized Majorana spinor  $\lambda$  describes the gluino field,  $\theta$  is the vacuum angle for the nonabelian gauge field  $G_{\mu\nu}$ , and  $iD_\mu\lambda = i\partial_\mu\lambda - g[A_\mu, \lambda]$ . This theory has an anomalous  $U(1)_R$  global symmetry,  $\lambda \rightarrow \exp(i\vartheta)\lambda$ , broken by instanton effects to  $Z_{2N_c}$ .

Various studies indicate the formation of a gluino condensate [7, 8], which spontaneously breaks<sup>1</sup> the discrete  $Z_{2N_c}$  symmetry to  $Z_2$ . Nonperturbative holomorphic calculations derive this result starting from an exact Wilsonian effective superpotential for  $N_f = N_c - 1$ , obtained [8, 10, 11]

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<sup>1</sup> Note however a suggestion to the contrary [9].

from supersymmetric and global symmetry constraints, assuming the existence of a nonperturbative, supersymmetric regulator. Successively integrating out massive flavors [12] results in an effective superpotential for  $N_f = 0$ . Differentiating with respect to the supersymmetric coupling  $\tau \equiv i4\pi/g_h^2 + \theta/2\pi$  then provides an exact formula [8] for the gluino condensate,  $\langle \lambda_h^a \lambda_h^a \rangle = 32\pi^2 \Lambda_h^3$ . Here, the subscript  $h$  denotes a “holomorphic” normalization [13] ( $1/g_h^2$  on the kinetic term) for a Weyl spinor, and  $\Lambda_h \equiv M \exp[2\pi i\tau/(3N_c)]$  is defined in terms of a holomorphic gauge coupling  $g_h^2$  defined at an ultraviolet scale  $M$  in the dimensional reduction scheme [8]. For a canonical field normalization, rescaling  $\lambda_h = g(M^2)\lambda$  requires the canonical coupling  $g$  to satisfy [14]  $8\pi^2/g_h^2(M^2) = 8\pi^2/g^2(M^2) + N_c \ln g^2(M^2)$ , due to the transformation’s anomalous Jacobian [13]. We can accordingly express the holomorphic result as

$$\frac{1}{2} \langle \bar{\lambda}^a \lambda^a \rangle = \frac{32\pi^2}{g^2(M^2)} \Lambda_h^3 = \frac{32\pi^2}{g^4(M^2)} \Lambda_1^3 \quad (2)$$

(with canonically normalized, Majorana spinors). The one-loop  $\beta$  function solution for  $g$  diverges at the “confinement” scale  $\Lambda_1 \equiv M \exp[-8\pi^2/(3N_c g^2(M^2))]$ .

Gap equation techniques, too, indicate a  $Z_{2N_c}$ -breaking condensate, signalled by a dynamical gluino mass. We write the gluino inverse propagator as  $A_\lambda(p^2) [\not{p} - \tilde{\Sigma}_\lambda(p^2)] i^{-1}$ , with the wave function factor  $A_\lambda(p^2)$  defined for renormalized fields so that  $A_\lambda(\mu^2) = 1$ , and with the dynamical mass  $\Sigma_\lambda(p^2) = A_\lambda(p^2) \tilde{\Sigma}_\lambda(p^2)$ . In a gauge with gluon propagator  $-i[g^{\mu\nu} + (\xi - 1)k^\mu k^\nu/k^2]/k^2$ , the gap equation in Euclidean space takes the form

$$\tilde{\Sigma}_\lambda(p^2) = C_2^{\text{adj}} g^2 (3 + \xi) \int \frac{dk^2}{16\pi^2} \frac{k^2}{\mathcal{M}^2} \frac{A_\lambda(\mathcal{M}^2)}{A_\lambda(m^2)} \frac{\tilde{\Sigma}_\lambda(k^2)}{k^2 + \tilde{\Sigma}_\lambda^2(k^2)}, \quad (3)$$

where  $\mathcal{M}$  and  $m$  are respectively the larger and smaller of  $k$  and  $p$ . (The angular integrations have been performed in four dimensions, after approximating  $A_\lambda([k - p]^2)$  by  $A_\lambda(\mathcal{M}^2)$ , and inserting an extra factor of  $A_\lambda(\mathcal{M}^2)$  in anticipation of the linearization discussed below.) The structure of this equation is similar to that for the quark in ordinary QCD [5, 3], differing only in that the group factor  $C_2^{\text{adj}} = N_c$  replaces  $C_2^{\text{fund}} = (N_c^2 - 1)/(2N_c)$ . We have neglected running of the coupling and of the gauge parameter [5], a crude approximation adequate to establish the existence of a symmetry-breaking solution in this theory and to set the stage for the presence of matter fields.

Here, the approximation requires an ultraviolet cutoff on the integral. When matter fields are included, an infrared fixed point will govern the transition and justify the neglect of running.

A nontrivial solution to Eq. (3) requires the coupling to exceed a critical value, near which the dynamical mass vanishes continuously [15]. This allows the critical coupling to be determined by analyzing the equation in the regime  $p \gg \tilde{\Sigma}_\lambda(p^2)$ , where loop momenta  $k \gg \tilde{\Sigma}_\lambda(k^2)$  dominate the integral and linearization in  $\tilde{\Sigma}_\lambda$  is a good approximation. We can then also neglect corrections to the massless renormalization group formula  $A_\lambda(p^2) \approx (\mu^2/p^2)^{\gamma_\lambda}$ , where the scaling exponent equals the gluino-field anomalous dimension

$$\gamma_\lambda = C_2^{\text{adj}} \xi \frac{g^2}{16\pi^2} . \quad (4)$$

We find solutions by [3] inserting a scaling form  $\tilde{\Sigma}_\lambda(p^2) \approx \tilde{\Sigma}_\lambda(\mu^2) (\mu^2/p^2)^{b_\lambda}$ . The scaling exponent  $b_\lambda$ , in leading approximation just (half) the gluino-mass anomalous dimension, obeys a  $\xi$ -independent equation to first order in  $g^2$ :

$$b_\lambda (1 - b_\lambda) = 3 C_2^{\text{adj}} \frac{g^2}{16\pi^2} + \mathcal{O}(g^4) . \quad (5)$$

A solution requires the right hand side of this equation to be large enough for  $b_\lambda$  to become complex [5], corresponding to the mass anomalous dimension reaching unity. The discrete global symmetry is predicted to break if the coupling exceeds the resulting critical minimum value,  $\alpha_{\text{cr}} \equiv g_{\text{cr}}^2/4\pi = \pi/(3C_2^{\text{adj}}) = \pi/(3N_c)$ . Higher order corrections to the kernel of the gap equation may be small enough not to affect this conclusion qualitatively. In nonsupersymmetric QCD, the next order corrections are less than 20% [3, 4].

In reality, the coupling runs; this effect can be incorporated in a WKB-like approximation [5] which predicts nonzero solutions at scales where the coupling exceeds the above critical value. Since the theory is asymptotically free and presumably confining, that will occur at momenta of order the confinement scale.<sup>2</sup> The symmetry is thus predicted to break, and the confinement scale sets the size of  $\tilde{\Sigma}_\lambda$  via the full nonlinear gap equation. Inserting this result into the operator product

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<sup>2</sup> The Wilsonian effective coupling has been argued to be controlled by an exact  $\beta$  function [16] for  $N_f = 0$ :  $dg/d\ln\mu = -(3N_c/16\pi^2) g^3/(1 - N_c g^2/(8\pi^2))$ . Well before reaching the singularity in this  $\beta$  function, the running coupling will reach the above  $g_{\text{cr}}$ . The  $\beta$  function's expansion parameter there is  $N_c \alpha_{\text{cr}}/(2\pi) = 1/6$ .

formula for the gluino condensate leads to the estimate

$$\langle \bar{\lambda}^a \lambda^a \rangle \approx 4(N_c^2 - 1) \int \frac{d^4 k}{(2\pi)^4} \frac{\tilde{\Sigma}_\lambda(k^2)}{k^2 + \tilde{\Sigma}_\lambda^2(k^2)} \frac{1}{A_\lambda(k^2)} \sim (N_c^2 - 1) \frac{\Lambda_1^3}{4\pi^2}. \quad (6)$$

The last rough estimate, neglecting logarithmic factors, arises from dimensional analysis with a nonzero solution to the gap equation, identifying the confinement scale with  $\Lambda_1$  in Eq. (2). For large  $N_c$ , we may write this estimate in the form

$$\langle \bar{\lambda}^a \lambda^a \rangle \sim \frac{1}{2} \left( \frac{N_c g^2(M^2)}{8\pi^2} \right)^2 \frac{32\pi^2}{g^4(M^2)} \Lambda_1^3, \quad (7)$$

which has the same  $N_c$  scaling (with fixed  $N_c g^2(M^2)$ ) as Eq. (2).

We note that a complete gap-equation prediction of symmetry breaking would require comparing the broken solution to the  $\tilde{\Sigma}_\lambda = 0$  solution. Establishing the former to be the ground state would require settling currently unresolved issues of gauge dependence, resulting from truncating the effective action or the infinite set of coupled Schwinger-Dyson equations. (Even in QCD, gap equation techniques face the same question.) Assuming the broken solution to be the ground state (with zero energy for unbroken supersymmetry), the gap equation and holomorphic approaches both predict that the discrete  $Z_{2N_c}$  symmetry spontaneously breaks to  $Z_2$  via a gluino condensate.

### 3 Supersymmetric QCD with $N_f$ Flavors

Including quark supermultiplets with  $N_f$  flavors provides more scope for comparing the two approaches. Again using component notation with auxiliary fields eliminated, we add  $N_f$  quark flavors to the Lagrangian, as Dirac spinors in the  $SU(N_c)$  fundamental representation; their left- and right-handed components are respectively associated with scalar superpartners,  $\phi$  and  $\tilde{\phi}^*$ :

$$\begin{aligned} \mathcal{L}_{SQCD} = & \mathcal{L}_{SYM} + \bar{\psi}(i\not{D} - m_0)\psi + |D_\mu \phi|^2 + |\tilde{D}_\mu \tilde{\phi}|^2 - m_0^2 (\phi^* \phi + \tilde{\phi} \tilde{\phi}^*) \\ & - \frac{g^2}{2} (\phi^* T^a \phi - \tilde{\phi} T^a \tilde{\phi}^*)^2 - ig\sqrt{2} (\phi^* T^a \bar{\lambda}^a P_L \psi + \tilde{\phi} T^a \bar{\lambda}^a P_R \psi - \text{h.c.}), \end{aligned} \quad (8)$$

where  $iD_\mu \equiv i\partial_\mu - gA_\mu^a T^a$ . In the limit of vanishing bare masses  $m_0 \rightarrow 0$ , the global symmetries are: a discrete parity symmetry,  $(\psi_L, \phi) \leftrightarrow (\psi_R, \tilde{\phi}^*)$ ; chiral  $SU(N_f) \times SU(N_f)$  with quarks and squarks rotated together; baryon number  $U(1)_B$  for quarks and squarks; and the anomaly-free subgroup

$U(1)_X$  of the gluino/squark  $U(1)_R$  and the quark/squark axial  $U(1)_A$ . Although these symmetries forbid perturbative mass generation, nonperturbative condensates could break the symmetries, generating fermion and scalar dynamical masses and mixing the left- and right- scalars. The scalar mixing would also require nonvanishing dynamical gluino mass, since  $U(1)_X$  combines gluino and axial rotations.

An important step in the nonperturbative study of supersymmetric theories was Witten's observation [17] that in a massive theory the difference between the number of bosonic and fermionic zero-energy states is conserved<sup>3</sup> (the Witten index), with a nonzero value ( $N_c$  for massive SQCD) implying unbroken supersymmetry. In that case, supersymmetric Ward identities for  $N_f > 0$  leave fermion condensates proportional to the bare mass. Specifically, for the gluino condensate the Konishi anomaly [19] gives  $\frac{1}{2} \langle \bar{\lambda} \lambda \rangle = (g^2/16\pi^2) m_0 \langle \phi \tilde{\phi} \rangle$ . For the quark condensate [20], each flavor  $i$  satisfies  $\langle \bar{\psi}_i \psi_i \rangle = m_0 \langle \phi_i^* \phi_i + \tilde{\phi}_i \tilde{\phi}_i^* \rangle$ . The massless theory thus forbids fermion condensates, assuming a well-defined, nonvanishing index at  $m_0 = 0$ , and assuming squark bilinear expectations are not too singular when  $m_0 \rightarrow 0$ .

The holomorphic effective action approach [6] provides a more complete description of SQCD with  $N_f$  flavors. Assuming the existence of a nonperturbative, supersymmetric regulator, it leads to a self-consistent picture for various values of  $N_f$ . Many physically distinct but (if  $N_f \geq N_c$ ) degenerate vacua form a moduli space, on which generically the chiral symmetry breaks, but at special points (when expectation values vanish) is preserved.

- For  $N_f > 3N_c$ , there is no asymptotic freedom, and the quarks, squarks, gluons and gluinos become noninteracting at large distances.
- For  $\frac{3}{2}N_c < N_f < 3N_c$ , the theory is asymptotically free and the coupling runs to a “superconformal” infrared fixed point. There is no confinement in this “nonabelian Coulomb phase”, and the global symmetries remain unbroken at the origin of moduli space (where the squark vacuum expectation values vanish). The spectrum is described by interacting, massless quarks, squarks, gluons, and gluinos.

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<sup>3</sup> For the exactly massless theory, zero modes can come in from infinity and leave the index ill-defined. This is avoidable [18] if  $N_f$  is an integer multiple of  $N_c$ .

- For  $N_c + 2 \leq N_f \leq \frac{3}{2}N_c$ , there is no confinement, but the gauge theory is strongly interacting in the infrared. The spectrum is best described in terms of massless, composite mesons and baryons of a local, infrared-free, dual gauge theory. At the origin of moduli space the theory leaves unbroken all the original global symmetries.
- For  $N_f = N_c$  or  $N_f = N_c + 1$ , the theory confines and the spectrum consists of massless composite particles corresponding to meson and baryon fields composed of the original matter fields. For  $N_f = N_c$ , either  $U(1)_B$  or chiral  $SU(N_f) \times SU(N_f)$  must break. For  $N_f = N_c + 1$ , there can be confinement without chiral symmetry breaking.
- For  $1 \leq N_f < N_c$ , a nonzero superpotential [11] lifts the degeneracy and fixes the ground state at arbitrarily large scalar expectation values (for  $m_0 \rightarrow 0$ ).

An important feature of these results is that for  $N_f > N_c$ , the global symmetries associated with massless quarks, squarks and gluinos can all remain unbroken.

We next use the gap equation framework to study the SQCD theory with matter. A previous investigation [21] concentrated on the quark and squark dynamical masses, neglecting the gluino condensate by assuming the matter fields to occupy a high dimensional representation. Here we will retain all condensates, to obtain a set of coupled gap equations for the quarks, gluinos, and squarks. The most attractive interactions occur in channels that preserve local  $SU(N_c)$  and parity, so we will not consider color-breaking condensates, or differing dynamical masses for the scalars  $\phi$  and  $\tilde{\phi}$  (Eq. 8). We will also exclude nonzero vacuum values for the squark fields. Supersymmetry requires equal bare masses for the quarks and squarks, which we take to vanish; but the dynamical masses can differ, since the component notation does not maintain manifest supersymmetry in off-shell Green functions.

For small enough coupling, the running will be determined by the (scheme-independent) two-loop  $\beta$  function [22]

$$\frac{dg}{d\ln\mu} = -\frac{g^3}{16\pi^2}(3N_c - N_f) - \frac{2g^5}{(16\pi^2)^2}(3N_c^2 - 2N_cN_f + N_f/N_c) + \dots, \quad (9)$$

which displays an infrared fixed point when  $3N_c/(2 - N_c^{-2}) < N_f < 3N_c$ . It occurs at

$$\alpha_* = 2\pi \frac{3N_c - N_f}{2N_cN_f - 3N_c^2 - N_f/N_c}, \quad (10)$$

a small value for  $N_f$  sufficiently close to  $3N_c$ . The coupling approaches this fixed point at scales below some intrinsic scale  $\Lambda$  governing the solution to Eq. (9). If the fixed point coupling exceeds a critical value, then the gap equations will exhibit nontrivial solutions. Since the dynamical mass vanishes continuously there,<sup>4</sup> the transition will be governed by the fixed point, and the running of the coupling may be neglected to first approximation. Furthermore, the gap equations may be linearized in the neighborhood of the transition.

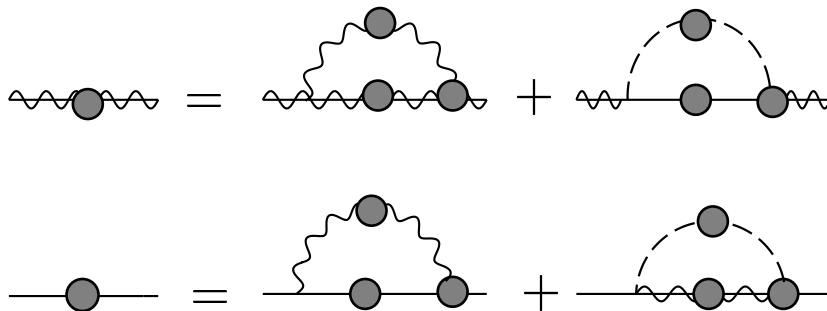


Figure 1: Gap equations for the gluino and for the quark masses. Solid lines are quarks, dashed lines are squarks, wiggled lines are gluons, and wiggled-on-solid lines are gluinos. Blobs symbolize the full propagators and vertex structures. The scalar exchange graphs give integrands proportional to higher powers of dynamical mass, and are dropped in the linearized approximation.

The gluino and quark gap equations are shown in Fig. 1. To lowest order in the kernel and in the linearized approximation relevant to the study of the transition and the determination of the critical coupling, they each receive a contribution only from the first graph (gluon emission and reabsorption). The gluino gap equation is simply the linearized version of Eq. (3), and the corresponding quark equation takes the form [3]

$$\tilde{\Sigma}_\psi(p^2) = g^2 C_2^{\text{fund}} (3 + \xi) \int \frac{dk^2}{16\pi^2} \frac{k^2}{\mathcal{M}^2} \frac{A_\psi(\mathcal{M}^2)}{A_\psi(m^2)} \frac{\tilde{\Sigma}_\psi(k^2)}{k^2}, \quad (11)$$

where  $g^2$  is now the value of the coupling at the infrared fixed point. For both, the intrinsic scale  $\Lambda$  is effectively an ultraviolet cutoff in this approximation: the functions  $\tilde{\Sigma}(p^2)$  steepen there from near-critical behavior  $\sim 1/p$ , to asymptotic behavior  $\sim 1/p^2$ . The gauge parameter is also in general a function of momentum, governed by a renormalization group equation. When  $g^2$  approaches its

<sup>4</sup> Ref. [15] argued that although the dynamical mass vanishes continuously, there are no light degrees of freedom in the symmetric phase and therefore the transition is not, strictly speaking, of second order.



infrared fixed point, the gauge parameter has one as well. We denote this fixed-point value simply by  $\xi$ , and in this approximation take it outside the integral. The wave function factors  $A(p^2)$  are computed in the massless theory, dropping corrections subleading in mass. They are defined to be unity at the renormalization scale  $\mu^2$ .

For the scalars, we obtain a gap equation for the diagonal mass  $\tilde{\Sigma}_\phi^2(p^2)$ , appearing in both  $\langle \phi(p) \phi^*(-p) \rangle$  and  $\langle \tilde{\phi}^*(p) \tilde{\phi}(-p) \rangle$ :

$$\begin{aligned}
& p^2 (A_\phi(p^2) - Z_\phi) + A_\phi(p^2) \tilde{\Sigma}_\phi^2(p^2) \\
&= C_2^{\text{fund}} \int \frac{dk^2}{16\pi^2} g^2 \left[ \frac{A_\phi^2(\mathcal{M}^2)}{A_\phi(k^2)} \left\{ 1 - \xi + (\xi - 3) \frac{m^2}{\mathcal{M}^2} \right\} \frac{k^2}{k^2 + \tilde{\Sigma}_\phi^2(k^2)} + (3 + \xi) A_\phi(\mathcal{M}^2) \frac{A_g(\mathcal{M}^2)}{A_g(k^2)} \right. \\
&\quad \left. - \frac{A_\phi(\mathcal{M}^2) A_\psi(\mathcal{M}^2)}{A_\psi(k^2)} \frac{2}{k^2 + \tilde{\Sigma}_\psi^2(k^2)} \left\{ k^2 + \frac{m^2}{p^2} (k^2 - p^2) - \tilde{\Sigma}_\lambda^2(\mathcal{M}^2) \cdot [1 + \text{sign}(k^2 - p^2)] \right\} \right]
\end{aligned} \tag{12}$$

where the wave function renormalization constant  $Z_\phi$  implements  $A_\phi(\mu^2) = 1$ , and  $A_g(k^2)$  represents the gluon wave function factor.<sup>5</sup> The dynamical mixing mass  $\tilde{\Sigma}_X^2(p^2)$  in  $\langle \tilde{\phi}(p) \phi(-p) \rangle$  satisfies

$$\begin{aligned}
\tilde{\Sigma}_X^2(p^2) &= C_2^{\text{fund}} \int \frac{dk^2}{16\pi^2} \frac{g^2}{\mathcal{M}^2} \left[ \frac{A_\phi(\mathcal{M}^2)}{A_\phi(m^2)} \tilde{\Sigma}_X^2(k^2) \left\{ (\xi + 1) \mathcal{M}^2 + (3 - \xi) m^2 \right\} \frac{k^2}{(k^2 + \tilde{\Sigma}_\phi^2(k^2))^2} \right. \\
&\quad \left. + 4 \frac{A_\phi(\mathcal{M}^2) A_\psi(\mathcal{M}^2)}{A_\phi(p^2) A_\psi(k^2)} \tilde{\Sigma}_\psi(k^2) \tilde{\Sigma}_\lambda(\mathcal{M}^2) \frac{k^2}{k^2 + \tilde{\Sigma}_\psi^2(k^2)} \right].
\end{aligned} \tag{13}$$

In Eqs. (12) and (13) we have dropped some contributions from diagrams with higher powers of mass insertions, as well as higher order contributions from diagonalizing the scalar propagator. Supersymmetry ensures cancellation of the quadratic divergences in the diagonal mass equation (12). After that cancellation, both equations can be fully linearized for momenta  $p$  large compared to  $\tilde{\Sigma}$ .

Then up to mass-suppressed corrections,  $A_\phi(p^2)$  is fixed by Eqn. (12), while  $A_\lambda(p^2) \approx (\mu^2/p^2)^{\gamma_\lambda}$  and  $A_\psi(p^2) \approx (\mu^2/p^2)^{\gamma_\psi}$ , where

$$\begin{aligned}
\gamma_\lambda &= \frac{g^2}{16\pi^2} (\xi C_2^{\text{adj}} + 2N_f C_{\text{fund}}) , \\
\gamma_\psi &= \frac{g^2}{16\pi^2} (\xi + 1) C_2^{\text{fund}} .
\end{aligned} \tag{14}$$

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<sup>5</sup> Here,  $g$  and  $\xi$  are in general functions of  $\mathcal{M}^2$ . As in the fermion equations, this dependence can be neglected if the integrals are dominated by an infrared fixed point.

Here  $C_{\text{fund}} = 1/2$ , and  $g^2$  and  $\xi$  are the fixed point values described above. Since the critical couplings depend on the wave function factors, the gluino critical coupling will differ from its value in the matter-free theory, even though in both cases gluon emission and reabsorption is the only leading contribution to the gap equation.

We focus initially on the quark and gluino equations. As described for the matterless theory, substituting the wave function factors of Eq. (14) into the gap equations, and setting the mass anomalous dimensions to unity, leads to critical couplings independent of  $\xi$ :

$$\begin{aligned}\alpha_{\text{cr},\lambda} &= \frac{\pi}{3 C_2^{\text{adj}} - 2N_f C_{\text{fund}}} = \frac{\pi}{3N_c - N_f}, \\ \alpha_{\text{cr},\psi} &= \frac{\pi}{2 C_2^{\text{fund}}} = \frac{\pi}{N_c - 1/N_c}.\end{aligned}\tag{15}$$

The choice of  $N_f$  determines whether the infrared coupling  $\alpha_*$  in Eq. (10) achieves these critical values. As  $N_f$  is reduced from  $3N_c$ ,  $\alpha_*$  increases from zero and first exceeds the quark critical coupling,  $\alpha_{\text{cr},\psi}$ , when  $N_f/N_c = (\frac{9}{4})\frac{1-(2/3)N_c^{-2}}{1-(3/4)N_c^{-2}} \approx 2.25$ . For this value of  $N_f/N_c$ ,  $\alpha_*$  does not yet reach the gluino critical coupling, leaving quark condensation to play the primary role here. Assuming validity of the approximations above, we can now outline the phase structure of SQCD as described by gap equations:

- For  $N_f > 3N_c$  there is no asymptotic freedom, in agreement with the holomorphic description.
- For  $2.25N_c < N_f < 3N_c$  there is an infrared fixed point at which the two-loop  $\beta$  function vanishes. For  $N_f$  close enough to  $3N_c$ , the fixed point coupling is small, keeping the coupling well below the critical values of Eq. (15). For any  $N_f$  above  $2.25N_c$ , the infrared coupling remains below both critical couplings, leaving the quark and gluino symmetries unbroken. Confinement does not set in and the theory remains in the nonabelian Coulomb phase, in agreement with the holomorphic prediction.
- For  $N_f < 2.25N_c$ , the infrared coupling (in the two-loop approximation) exceeds the quark critical coupling. If higher order effects may be neglected, the gap equation then indicates that quarks develop a dynamical mass which vanishes continuously at the transition. The quarks decouple from the  $\beta$  function at this mass scale, eliminating the fixed point and causing  $g$  to run to the gluinos' (larger) critical coupling. Gluino and scalar dynamical masses are then generated at essentially the

same scale as the quark mass. Below these mass scales, decoupling allows  $g$  to continue running to confining values. The global chiral and  $U(1)_X$  symmetries are thus predicted to break in this  $N_f$  range. The supersymmetric Ward identity then requires that supersymmetry breaks as well.

Holomorphic methods, by contrast, predict the nonabelian Coulomb phase to remain down to  $N_f = \frac{3}{2}N_c$ , and allow preservation of the global symmetries without confinement even down to  $N_f = N_c + 1$ .

These patterns also control the scalar gap equation solutions, which depend upon the fermion solutions. The inhomogeneous scalar equations do not admit vanishing solutions when the quark and gluino dynamical masses are nonzero [21]. In that case, the scalar solutions should be of comparable scales. The connection between scalar and fermion mass solutions can be understood from the global symmetries: a nonzero off-diagonal mass  $\tilde{\Sigma}_X^2(p^2)$  breaks both  $U(1)_X$  and  $SU(N_f) \times SU(N_f)$ , and thus requires corresponding gluino and quark masses. The diagonal mass  $\tilde{\Sigma}_\phi^2(p^2)$  breaks no chiral or  $U(1)$  symmetries, but is not forced to vanish unless the quark mass does and supersymmetry is unbroken.

Checking that these symmetry breaking solutions of the gap equation describe the ground state would require showing that the resulting vacuum energy is lower than that of the chirally symmetric solution. (For this to be the case the symmetric solution, as well as the broken one, would need to break supersymmetry.) For SQCD with matter, suggestive calculations [21, 23] do indicate lower energy for the symmetry breaking vacuum, although these are gauge dependent and neglect squark and/or gluino dynamical masses.

## 4 Conclusion

In pure SQCD without matter fields, various approaches indicate that the discrete global symmetry  $Z_{2N_c}$  breaks spontaneously to  $Z_2$  through the formation of a gluino condensate. The theory confines and supersymmetry presumably remains unbroken. In the gap equation, the lowest order kernel, single gluon exchange, provides an attractive force capable of condensate formation. Whether the broken solution represents the ground state of the theory, however, remains unclear. Assuming

this to be the case, the gap equation and holomorphic effective action techniques lead to the same conclusion.

For SQCD with matter multiplets, gluon exchange alone again gives the leading contribution to the linearized gap equation governing the transition. At this order, the gap equation (with gauge coupling governed by the two-loop  $\beta$  function) indicates that spontaneous breaking of the global symmetries occurs for  $N_f$  below a critical value near  $2.25N_c$ . This transitional value corresponds to the infrared fixed point coupling exceeding the critical strength necessary for quark condensation. If we instead determine the fixed point coupling from the three-loop  $\beta$  function (in the DR scheme) [22], the corresponding critical  $N_f \approx 2.08N_c$ , only an 8% shift in this renormalization scheme. Examining the size of higher order terms in the kernel of the quark gap equation, corresponding to the mass anomalous dimension at two loops, would give another check on the validity of these approximations. In nonsupersymmetric theories, such kernel corrections are of order 20% [3, 4]. With these caveats, for  $N_f$  just below the critical value the quark mass is nonzero but small, justifying the linearization.

At scales below their predicted mass the quarks decouple, eliminating the infrared fixed point so that the coupling increases to the gluino critical value at essentially the same scale. The gluinos, too, thus condense at the quark mass scale. Squark masses are induced by the fermion masses, and are also of the same scale. The supersymmetric Ward identity then predicts that supersymmetry also breaks in this regime. An effective theory with Goldstone bosons and Goldstinos emerges at low energies. All this is in contrast with the picture emerging from holomorphic effective action techniques, where the nonabelian Coulomb phase is argued to persist down to  $N_f = \frac{3}{2}N_c$  and the global symmetries to remain unbroken (at the origin of moduli space) down to  $N_f = N_c + 1$ .

Of course, as already stressed, gap equation solutions in general have not been shown to correspond to ground states. (Supersymmetric constraints on ground state energies may be important in this connection.) Settling the question involves still-unresolved issues of gauge invariance and interpretation of the effective potential. Furthermore, truncating kernels at one loop, and the  $\beta$  function at two loops, seems quantitatively less reliable than in nonsupersymmetric theories. The

kernel truncation, together with the use of component notation (Wess-Zumino gauge), may also in effect explicitly break supersymmetry. The latter possibility can be checked by analyzing a set of gap equations in the manifestly supersymmetric superspace formalism [24]. A recent study of this problem [25] indicates that symmetry-breaking solutions could exist, although infrared divergences in the formalism have so far obscured the analysis.

The possible discrepancy between gap equation analyses and results based on holomorphic effective action techniques and index theorems is noteworthy and invites further study. Efforts to reconcile them may help deepen our insight into the behavior of strongly coupled gauge theories.

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